A Posteriori Error Estimator for Photonic Crystals

11 Nov 2005

Outline

Introduction
- Photonic Crystals
- Maxwell Equations
- TH and TE cases
- Floquet transform

TE case
- TE case

A Posteriori Error Estimator
- Introduction

Numerical Results

Conclusion

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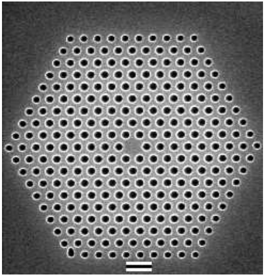
What is a Photonic Crystal?



"Photonic crystals are periodic dielectric structures [...] that are designed to affect the propagation of electromagnetic waves (EM) [...]. The absence of allowed propagating EM modes inside the structures, in a range of wavelengths is called a photonic band gap..." (Wikipedia Encyclopedia)

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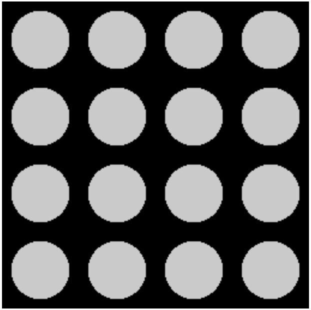

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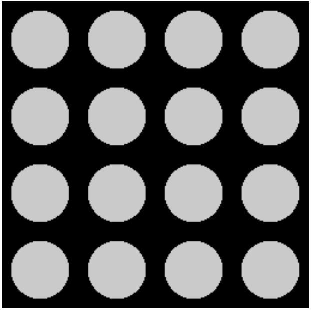

Periodic structure is composed of identical square pieces covering all the real plane and invariant under translation along the z-coordinate.

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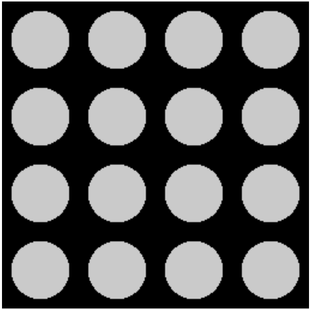

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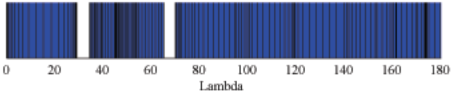

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Maxwell Equations

$$\begin{cases} \nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \\ \nabla \times \mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}, \\ \nabla \cdot \mathbf{B} = 0, \\ \nabla \cdot \mathbf{D} = 0, \end{cases}$$

E macroscopic electric field
H macroscopic magentic field
D displacement field
B magnetic induction field
c speed of light

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Constitutive Relations:

$$\begin{cases} \mathbf{D} = \varepsilon \mathbf{E}, \\ \mathbf{B} = \mu \mathbf{H}. \end{cases}$$

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Fourier Transform in Time Domain:

$$\mathbf{E}(\mathbf{x}, t) = e^{i\omega t}\mathbf{E}(\mathbf{x}),$$
$$\mathbf{H}(\mathbf{x}, t) = e^{i\omega t}\mathbf{H}(\mathbf{x})$$

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ω frequency

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$$\mu = 1,$$
$$\varepsilon = \begin{pmatrix} \epsilon_{11} & \epsilon_{12} & 0 \\ \epsilon_{21} & \epsilon_{22} & 0 \\ 0 & 0 & \epsilon_{33} \end{pmatrix},$$
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TH Case:
$\mathbf{E} = (0, 0, u(x, y))$
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$$\nabla \times (\varepsilon^{-1} \nabla \times \mathbf{H}) - \frac{\omega^2}{c^2}\mathbf{H} = 0.$$

$$\nabla \cdot (M \nabla u) + \frac{\omega^2}{c^2} u = 0, \quad M = \frac{1}{\epsilon_{11}\epsilon_{22} - \epsilon_{12}^2} \begin{pmatrix} \epsilon_{11} & \epsilon_{12} \\ \epsilon_{21} & \epsilon_{22} \end{pmatrix}.$$

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$$\left\{\begin{array}{l}\nabla\times\mathbf{E}=-\frac{i\omega}{c}\mu\mathbf{H},\\ \nabla\times\mathbf{H}=\frac{i\omega}{c}\varepsilon\mathbf{E},\\ \nabla\cdot\mu\mathbf{H}=0,\\ \nabla\cdot\varepsilon\mathbf{E}=0.\end{array}\right.$$

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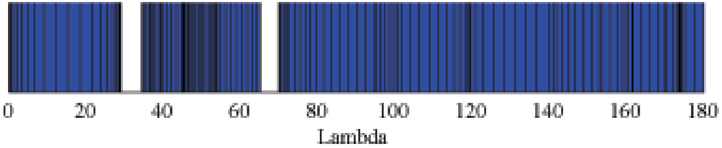


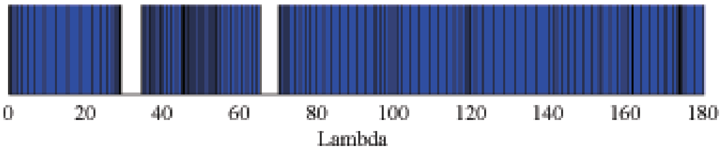
Problem: the spectrum is not discrete

$$g \in L^2(\mathbb{R}^2) \quad (\mathcal{F}g)(\alpha, \mathbf{x}) = e^{-i\alpha\cdot\mathbf{x}} \sum_{\mathbf{n}\in\mathbb{Z}^2} g(\mathbf{x}-\mathbf{n})e^{i\alpha\cdot n}$$

$$\alpha \in [-\pi, \pi]^2, x \in [0, 1]^2$$

$\Rightarrow$Now the domain of x is a torus and α is a parameter.

Floquet transform



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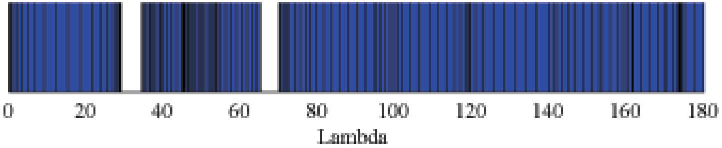

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$$(\mathcal{F}g)(\alpha, \mathbf{x}) = e^{-i\alpha\cdot\mathbf{x}} \sum_{\mathbf{n}\in\mathbb{Z}^2} g(\mathbf{x}-\mathbf{n})e^{i\alpha\cdot n} \qquad \alpha \in [-\pi, \pi]^2.$$

Lemma: $\mathcal{F}(\nabla g) = (\nabla + i\alpha)\mathcal{F}g.$

$$\text{TH: } \nabla^2 u + \frac{\omega^2}{c^2}\epsilon_{33} u = 0, \quad x \in \mathbb{T}, \alpha \in [-\pi, \pi]^2,$$

$$\text{TE: } \nabla \cdot (M\nabla u) + \frac{\omega^2}{c^2} u = 0, \quad x \in \mathbb{T}, \alpha \in [-\pi, \pi]^2.$$

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$$(\mathcal{F}g)(\alpha, \mathbf{x}) = e^{-i\alpha\cdot\mathbf{x}} \sum_{\mathbf{n}\in\mathbb{Z}^2} g(\mathbf{x} - \mathbf{n}) e^{i\alpha\cdot n} \qquad \alpha \in [-\pi, \pi]^2.$$

Lemma: $\mathcal{F}(\nabla g) = (\nabla + i\alpha)\mathcal{F}g.$

TH: $(\nabla + i\alpha)\cdot(\nabla + i\alpha)u_\alpha + \frac{\omega^2}{c^2}\epsilon_{33}u_\alpha = 0, \quad x \in \mathbb{T}, \alpha \in [-\pi, \pi]^2,$

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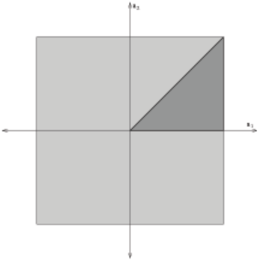

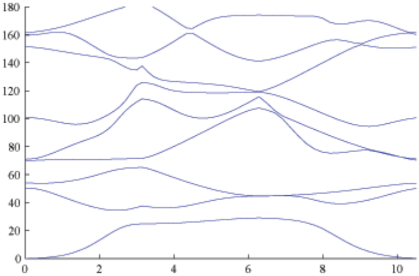

Outline

Introduction
Photonic Crystals
Maxwell Equations
TH and TE cases
Floquet transform

TE case
TE case

A Posteriori Error Estimator
Introduction

Numerical Results

Conclusion

Weak Form

$$u_\alpha \in H^1(\mathbb{T}, \mathbb{C}), -(\nabla + i\alpha) \cdot M(\nabla + i\alpha)u_\alpha = \lambda u_\alpha,$$

$$\lambda = \lambda(\alpha), \alpha \in [-\pi, \pi]^2$$

$$\int_{\mathbb{T}} M(\nabla + i\alpha)u_\alpha \cdot (\nabla - i\alpha)\overline{v}\, d\mathbf{t} = \int_{\mathbb{T}} \lambda u_\alpha \overline{v}\, d\mathbf{t}, \forall v \in H^1(\mathbb{T}, \mathbb{C})$$

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$$a_\alpha(u_\alpha, v) = \int_{\mathbb{T}} \lambda u_\alpha \overline{v}\, d\mathbf{t}, \forall v \in H^1(\mathbb{T},\mathbb{C})$$

The Spectrum is Discrete and Real

Properties of $a_\alpha(\cdot,\cdot)$:

- Hermitian
- Coercive: $a_\alpha(u,u) + K\|u\|^2_{0,\mathbb{T}} \geq \frac{1}{2}\|u\|^2_{1,\varepsilon,\mathbb{T}}, \quad \forall u \in H^1(\mathbb{T},\mathbb{C})$
- Continuous

Solution Operator:

$$a_\alpha(Tf, v) = (f, v)_{0,\mathbb{T}} \quad \forall v \in H^1(\mathbb{T},\mathbb{C}).$$

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What is a Posteriori Error Estimator?

$$a_\alpha(u,v) = \lambda(u,v), \qquad \text{(Continuous)}$$
$$a_\alpha(u_h,v_h) = \lambda_h(u_h,v_h), \qquad \text{(Discrete)}$$

Definitions:

- $\mathfrak{T}_h$ (Mesh), $V_h \subset H^1(\mathbb{T},\mathbb{C})$ (Finite Polynomial Space)
- $e_h := u - u_h$ (Error for Eigenfunctions)
- $\gamma_h := \lambda - \lambda_h$ (Error for Eigenvalues)
- $\eta(\lambda_h,u_h) = \left(\sum_{\tau\in\mathfrak{T}_h} \eta_\tau^2(\lambda_h,u_h)\right)^{1/2}$ (Error Estimator)

Properties:

1. $\|e_h\|_1 \le C\eta$ (Reliability)
2. $|\gamma_h| \le C\eta^2$ (Reliability)
3. $\eta \le C\|e_h\|_1 + h.o.t.$ (Efficiency)

All constant C are independent of h

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Why a Posteriori Error Estimator for TE case?

Reasons:

- Optimize Computational Power
- Global Regularity $H^{1+1/2-\beta}(\mathbb{T})$, $\beta > 0$
- Solutions could oscillate a lot,
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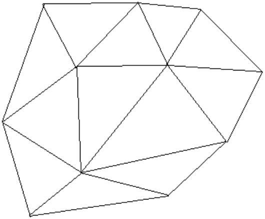
Residuals

$$\eta = \left(\sum_{\tau \in \mathfrak{T}_h} \eta_\tau^2(\lambda_h, u_h) \right)^{1/2},$$

$$R_I(u_h, \lambda_h)|_\tau := |(\nabla + i\alpha) \cdot m(\nabla + i\alpha)u_h + \lambda_h u_h|,$$



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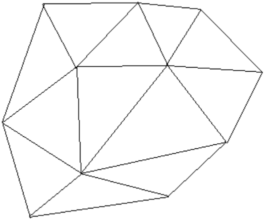

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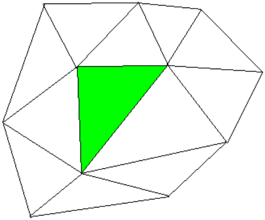

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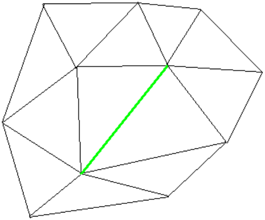

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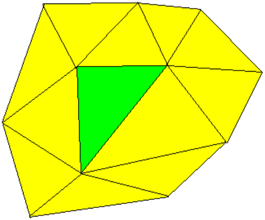

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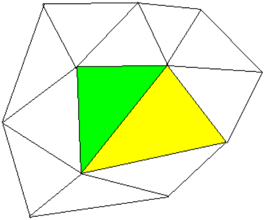

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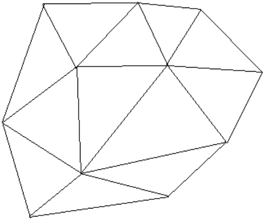

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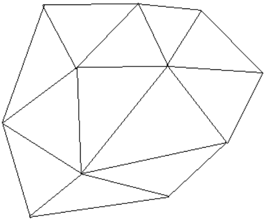

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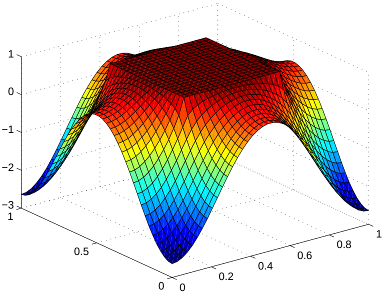
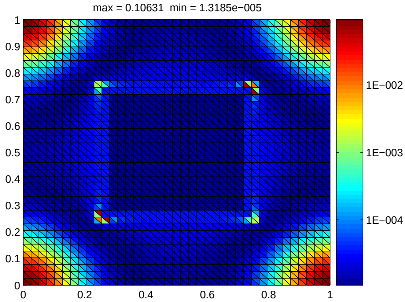
Numerical Results





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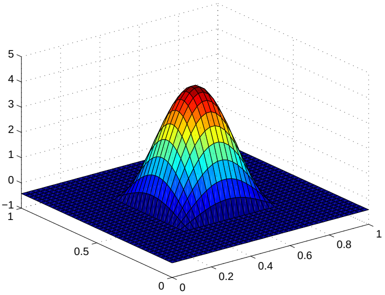

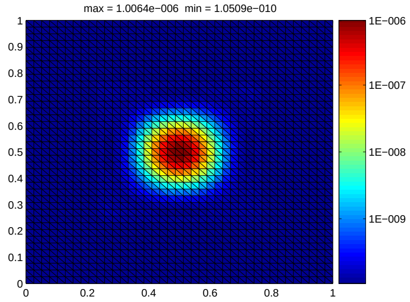

Further

- Iterative Methods (e.q. Multigrid)
- Crystals with Defects
- 3D Crystals
- Solver (e.g. Subspace Iteration)
- Robust Estimator for the jumping ratio.

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Global Regularity

A Model Problem: $-\nabla \cdot m \nabla u = f, \quad \forall f \in L^2(\mathbb{T}),$
Interface Conditions:

$$u_1 = u_2, \qquad \text{on } \Gamma \quad \text{(Continuity)}$$

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A Simpler Case: $-\nabla \cdot m \nabla u = \lambda u, \quad u \in H^1(\mathbb{T}),$
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Global Regularity

A Model Problem: $-\nabla \cdot m\nabla u = f, \quad \forall f \in L^2(\mathbb{T}),$

Interface Conditions:

$$u_1 = u_2, \qquad \text{on } \Gamma \quad \text{(Continuity)}$$

$$m_1 \frac{\partial u_1}{n_1} = -m_2 \frac{\partial u_2}{n_2}, \quad \text{on } \Gamma \quad \text{(Interface)}$$

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